

Effective constraint potential in lattice Weinberg - Salam model

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Abstract

We investigate lattice Weinberg - Salam model without fermions for the value of the Weinberg angle $\theta_W \sim 30^\circ$, and bare fine structure constant around $\alpha \sim \frac{1}{150}$. We consider the value of the scalar self coupling corresponding to bare Higgs mass around 150 GeV. The effective constraint potential for the zero momentum scalar field is used in order to investigate phenomena existing in the vicinity of the phase transition between the physical Higgs phase and the unphysical symmetric phase of the lattice model. This is the region of the phase diagram, where the continuum physics is to be approached. We compare the above mentioned effective potential (calculated in selected gauges) with the effective potential for the value of the scalar field at a fixed space - time point. We also calculate the renormalized fine structure constant using the correlator of Polyakov lines and compare it with the one - loop perturbative estimate.

1 Introduction

In lattice Electroweak theory the importance of the vicinity of the phase transition between the physical Higgs phase and the unphysical symmetric phase is related to the fact that this is the region of the phase diagram, where the continuum physics is to be approached. During the early studies of lattice Weinberg - Salam model it was recognized that the physics of the mentioned transition is intimately related to the way the continuum physics arises within the lattice model. Namely, in [1] it was suggested that the appearance of the second order phase transition leads to the conventional picture: It is possible to increase infinitely the ultraviolet cutoff along the line of constant physics corresponding to realistic values of renormalized couplings. At the same time,

according to [1] the first order phase transition would lead to another picture: the line of constant physics intersects the phase transition line at a certain value of the Ultraviolet cutoff Λ_c that is in this case the maximal possible value of the cutoff in the theory for the given values of renormalized couplings. It is worth mentioning that the first order phase transition was shown to take place at unphysical values of couplings [1]. However, at physical values of coupling constants two state signal was not found that means that we may deal either with the weak first order phase transition or with the second order phase transition.

Yet another possibility was suggested in [2]: the transition might appear to be a crossover. Starting from the physical Higgs phase and moving towards the symmetric phase one observes the increase of Nambu monopole [3, 4] density. (These objects are, in essence, the embryos of the symmetric phase within the Higgs phase.) At a certain point on the phase diagram the average distance between these embryos becomes of the order of their size. Further movement towards the symmetric phase leads to the point, where the only minimum of the ultraviolet effective constraint potential is at $\phi = 0$ while within the Higgs phase it has minimum at nonzero ϕ (see the next section for the definition of the potential). At this point, however, the Z - boson mass as well as the Higgs boson mass does not vanish (both masses are defined in the Unitary gauge when the scalar field is real and not negative). Then one can move further with the increase of the cutoff, and there should exist the point, where the transition to the true symmetric phase occurs (in this phase the gauge boson masses must vanish). According to [2] the vicinity of the phase transition, where the Nambu monopoles dominate, is called the fluctuational region. The possibility to describe the continuum Electroweak physics within the fluctuational region is questionable due to the Nambu monopoles that are supposed to give unexpected contributions to the physical observables. In particular, the transition might be a crossover (while the tree level perturbative effective action predicts the second order phase transition).

We would like to suggest a further analogy with the superconductor theory. Namely, for the second order superconductors in the presence of the external magnetic field there exist several pseudocritical lines on the phase diagram. First, when the value of the magnetic field achieves the value H_{c1} the Abrikosov vortices are formed. These objects are the embryos of the normal phase within the superconducting one. The mixed phase is formed, where the lattice of Abrikosov vortices exists within the superconductor.

Next, when the value of magnetic field achieves the pseudocritical value H_{c2} , the mixed phase is transformed to the normal phase. We suppose that the mixed phase of the second order superconductors is similar to the fluctuational region of the lattice Electroweak theory mentioned above. Of course, in our case we do not have any external field and the Z - vortices and Nambu monopoles arise spontaneously. Therefore, it is necessary to take care when applying the given analogy.

It is worth mentioning that the effective Abelian gauge model appears within the Weinberg - Salam model with the Z - boson playing the role of the Abelian gauge field. The second order superconductor appears within the Ginzburg - Landau model for $M_H > M_Z$. Therefore for $M_H > M_Z$ the Weinberg - Salam model may be similar to the second order superconductors (and not to the first order superconductors). For this reason, we do not expect, in particular, the appearance of the first order phase transition for $M_H > M_Z$. (The first order phase transition takes place for the first order superconductors.)

In the present paper we proceed with the research of [2] and consider the fluctuational region more carefully. The measurements were performed at much more different points in the vicinity of the transition than it was done in [2]. However, as a price for this we simulate the system on smaller lattices. Our main results reported here are obtained on the lattice $8^3 \times 16$ while in [2] the lattices up to the size $20^3 \times 24$ were used. As an additional device for the investigation we use the effective potential for the zero - momentum scalar field. In order to consider such a potential the gauge is to be fixed. We consider two different ways to fix the gauge and investigate the resulting effective potentials. It is shown, that one of the given potentials changes its form at the point γ_c , where the mentioned above ultraviolet effective potential changes its form. The other potential changes its form at the value γ'_c different from γ_c . This contradicts with the conventional picture that is based on the perturbation theory and implies that the scalar field condensates defined in different gauges vanish at the same point of the phase diagram. Thus the hypothesis that the given transition is a crossover is confirmed (at least on the lattices of considered sizes): different quantities change their behavior at different points on the phase diagram. However, the perturbation theory does not appear to be completely useless in the vicinity of the transition. Namely, we calculate the renormalized fine structure constant using the methods different from that of [2] and obtain a surprising coincidence with the 1 - loop estimate.

The paper is organized as follows. In Section 2 we consider the definition of the effective potentials under consideration. In Section 3 we consider the details of lattice regularized Weinberg - Salam model. In Section 4 we list our numerical results. In section 5 we discuss the obtained numerical results. Throughout the paper the notations of differential forms on the lattice are used (for their definition see, for example, [5]).

2 Infrared effective potential

In ordinary lattice scalar field theory (the real scalar field h_x is defined on the lattice points x) there exist several definitions of effective constraint potentials. Namely, one may consider the ultraviolet potential

$$\exp(-V^{u-v}(\phi)) = \langle \delta(\phi - h_x) \rangle \quad (1)$$

Also it is possible to consider the infrared potential

$$\exp(-V^{i-r}(\phi)) = \langle \delta(\phi - |\frac{1}{N} \sum_x h_x|) \rangle, \quad (2)$$

where N is the number of lattice points.

In principle, it is expected that the given potentials have nontrivial minima at nonzero ϕ in the broken phase, where the scalar field is condensed. However, in a more complicated model this statement is questionable because the value ϕ_m , at which the infrared potential has its minimum, is an infrared quantity while potential (1) is at a first look an ultraviolet quantity.

In the lattice theory for the complex scalar field charged with respect to lattice $U(1)$ gauge field $Z \in (-\pi; \pi]$ there are several complications. After fixing the gauge $H = \begin{pmatrix} h \\ 0 \end{pmatrix}$, $h \in C$, where H is the scalar doublet, the lattice Weinberg - Salam model becomes such a lattice gauge - Higgs model with the complex scalar field charged with respect to the Z - boson that plays now the role of the $U(1)$ gauge field.

Now (2) is not gauge invariant and, therefore, has the only minimum at $\phi = 0$ everywhere. There exist also naive gauge invariant version of the infrared potential:

$$\exp(-V'_{i-r}(\phi)) = \langle \delta(\phi - \frac{1}{N} \sum_x |h_x|) \rangle, \quad (3)$$

However, this potential, obviously, has the only minimum at $\phi \neq 0$ in both phases.

Instead of (3) we can consider potential (2) at a fixed gauge. Actually, (3) is equivalent to (2) for the version of the Unitary gauge adopted in [2]: $h_x \in R, h_x \geq 0$.

If the Unitary gauge is fixed using only the condition $h_x \in R$, the Z_2 gauge degrees of freedom remain: $h_x \rightarrow (-1)^{n_x} h_x, Z \rightarrow [Z + \pi dn] \bmod 2\pi$. This remaining gauge freedom is to be the subject of the further gauge fixing. The simplest choice here is $h_x > 0$. However, this choice does not lead to the effective potential sensitive to the transition between the two phases.

The other possible choice is minimization of

$$\sum_{links} (1 - \cos Z) \rightarrow min \quad (4)$$

with respect to the mentioned Z_2 transformations. Further we refer to this gauge as to the Z - version of Unitary gauge and refer to the corresponding effective potential (2) as to UZ potential.

Yet another way to define the Unitary gauge with $h_x \in R$ is to minimize the divergence of Z with respect to the remaining Z_2 transformations:

$$\sum_x [\delta Z]^2 \rightarrow min \quad (5)$$

Further we refer to this gauge as to the DZ - version of Unitary gauge and refer to the corresponding effective potential (2) as to UDZ potential. It will be shown that the infrared potential in this gauge has the only minimum at $\phi = 0$ in the region of the phase diagram, where potential (1) also has the only minimum at $\phi = 0$. In the region of the phase diagram, where the ultraviolet potential has its minimum at nonzero ϕ , the infrared potential (2) has its minimum at nonzero ϕ as well.

In principle, there exists also the possibility to consider the infrared effective potential for the scalar field surrounded by the Z - boson cloud (the Dirac construction [7]) that is gauge invariant by definition. However, the corresponding numerical procedure is rather time consuming, especially on large lattices. Therefore, at the present moment we do not consider such a construction.

3 The lattice model under investigation

We consider lattice Weinberg - Salam model without fermions. The partition function has the form:

$$Z = \int DHD\Gamma \exp(-A(\Gamma, H)) \quad (6)$$

Here $A(\Gamma, H)$ is the action for the scalar doublet H and the gauge field $\Gamma = U \otimes e^{i\theta} \in SU(2) \otimes U(1)$:

$$\begin{aligned} A(\Gamma, H) = & \beta \sum_{\text{plaquettes}} \left((1 - \frac{1}{2} \text{Tr } U_p) + \frac{1}{\text{tg}^2 \theta_W} (1 - \cos \theta_p) \right) + \\ & -\gamma \sum_{xy} \text{Re}(H_x^+ U_{xy} e^{i\theta_{xy}} H_y) + \sum_x (|H_x|^2 + \lambda(|H_x|^2 - 1)^2), \quad (7) \end{aligned}$$

The action can be rewritten as follows:

$$\begin{aligned} A(\Gamma, H) = & \beta \sum_{\text{plaquettes}} \left((1 - \frac{1}{2} \text{Tr } U_p) + \frac{1}{\text{tg}^2 \theta_W} (1 - \cos \theta_p) \right) + \\ & + \frac{\gamma}{2} \sum_{xy} |H_x - U_{xy} e^{i\theta_{xy}} H_y|^2 + \sum_x (|H_x|^2 (1 - 2\lambda - 4\gamma) + \lambda |H_x|^4) \quad (8) \end{aligned}$$

Now we easily derive expressions for the tree level vacuum expectation value v of $|H_x|$, the lattice Higgs boson mass $m_H = M_H a$, the lattice Z - boson mass $m_Z = M_Z a$, and the critical value γ_c :

$$\begin{aligned} v &= \sqrt{2 \frac{\gamma - \gamma_c}{\lambda}} \\ m_H &= v \sqrt{\frac{8\lambda}{\gamma}} \\ m_Z &= v \sqrt{\frac{\gamma}{\beta \cos^2 \theta_W}} \\ \gamma_c^{(0)} &= \frac{1 - 2\lambda}{4} \quad (9) \end{aligned}$$

After fixing the gauge $H = \begin{pmatrix} h \\ 0 \end{pmatrix}$, $h \in C$, where H is the scalar doublet, the lattice Weinberg - Salam model becomes the lattice gauge - Higgs model with the scalar field charged with respect to the Z - boson that plays now

the role of the $U(1)$ gauge field. Natural definition of the Z - boson field is $Z = \text{Arg}[U_{11}e^{i\theta}]$. Next, after fixing the Unitary gauge the field h becomes real. However, the Z_2 gauge ambiguity remains: $h_x \rightarrow (-1)^{n_x} h_x$, $Z \rightarrow [Z + \pi dn] \bmod 2\pi$.

The tree level approximation gives for the effective constraint potential (1):

$$V^{u-v}(\phi) = -3 \log \phi + \frac{\gamma}{2G_{m_H}(0)}(\phi - v)^2 \quad (10)$$

Here we encounter the lattice volume N and the value of lattice Yukawa potential at zero distance $G_{m_H}(0) = \frac{1}{N} \sum_p (4 \sin^2 p/2 + m_H^2)^{-1}$. Even on the infinite lattice this value remains finite if m_H is nonzero: $G_{m_H}(0) < 1/m_H^2$. This means that the ultraviolet fluctuations $\delta\phi = \sqrt{G_{m_H}(0)}/\gamma$ expressed in lattice units remain finite. When the physical volume of the lattice $Na^4 \gg M_H^{-4}$ is kept constant while the lattice spacing tends to zero and, consequently, $m_H = M_H a \rightarrow 0$ (here M_H is the Higgs mass in GeV, a is the lattice spacing) the value of $G_{m_H}(0)$ remains finite. Thus we recover the usual prediction of the continuum theory $\delta\phi^{phys} = \sqrt{G_{M_H}^{phys}(0)}/\gamma \sim \Lambda = \frac{\pi}{a}$, where ϕ^{phys} is the scalar field expressed in physical units while $G_{M_H}^{phys}(x)$ is its propagator in physical units (here in lattice regularization, though).

The tree level approximation gives for the infrared effective constraint potential (2):

$$V^{i-r}(\phi) = N\lambda(\phi^2 - v^2)^2 \quad (11)$$

This expression corresponds also to mean field approximation. Here v is the same as for the ultraviolet potential. Unlike the ultraviolet effective potential, however, the fluctuations of ϕ decrease fast with the increase of the lattice size. When the physical volume \mathcal{V} of the lattice $\mathcal{V} = Na^4$ is kept constant while the lattice spacing tends to zero and, consequently, $m = M_H a \rightarrow 0$ (here M_H is the Higgs mass in GeV, a is the lattice spacing) the fluctuations of ϕ in lattice units tend to zero while fluctuations in physical units $\sim 1/\sqrt{8\lambda v_{phys}^2} \mathcal{V}$ remain finite. Thus in mean field approximation the nontrivial minimum of the infrared constraint effective potential appears at the same value of the field as the nontrivial minimum of the ultraviolet effective potential. Taking into account loop corrections one would come, in principle, to the expression for the infrared effective constraint potential in the form of Coleman - Weinberg. (See, for example, [6], where the finite temperature version of the potential was given.)

4 Numerical results

We investigated numerically the system at $\beta = 12$, $\lambda = 0.0025$, $\theta_W = 30^\circ$. Our results were obtained mainly on the lattice $8^3 \times 16$. According to the previous results [2] at these values of couplings in the vicinity of the transition between the two phases the Higgs boson mass is around 150 GeV while bare fine structure constant is $\sim 1/150$. In [2] the transition point was localized at the point, where the ultraviolet effective constraint potential (described in Section 2 of the present paper) loses the nontrivial minimum at nonzero value of ϕ . Namely, at $\gamma > \gamma_c = 0.26 \pm 0.001$ the value of v calculated using this type of effective potential is nonzero while for $\gamma \leq \gamma_c$ the value of v vanishes. However, there are also two other selected points, denoted in [2] by γ_{c0} and γ_{c2} .

At γ_{c0} the dependence of the lattice Z - boson mass on γ gives $m_Z = 0$. The linear fit to the Z - boson mass calculated in [2] indicates that $\gamma_{c0} = 0.252 \pm 0.001$. However, we do not exclude that the Z - boson mass may vanish at the values of γ larger than 0.252. Actually, in [2] the nonzero Z - boson mass was obtained for $\gamma \geq 0.258$.

At $\gamma_{c2} \sim 0.262$ the average distance between Nambu monopoles becomes compared to their size. So, the fluctuational region is localized between γ_{c0} and γ_{c2} . Within this region it is expected that the perturbation theory does not work well and nonperturbative phenomena become important.

4.1 Simulation details

The model is simulated in Unitary gauge with the signs of h unfixed. Therefore, the Z_2 gauge freedom remains (together with the Electromagnetic $U(1)$). In order to simulate the system the Metropolis algorithm is used. The new suggested value of the gauge field is obtained via the (right) multiplication of the old gauge field at the given link by the $SU(2) \times U(1)$ random matrix. The values of this matrix are distributed randomly (with Gauss distribution) around unity. Norm of the Gauss distribution is tuned automatically in order to keep acceptance rate around 0.5. The new suggested value of the scalar field h is obtained adding to the old one the value δh distributed randomly (with Gauss distribution) around zero. The norm of this distribution (different from the norm used for the gauge fields) is also tuned in order to keep acceptance rate around 0.5. Each step of Metropolis procedure contains suggestions of new fields at all points and links of the lattice. The

procedure starts from the zero values of the scalar fields and the values of the gauge fields equal to unity (cold start). At each considered value of γ several independent processes run (up to 100 processes). Out of the interval $\gamma \in [0.2575; 0.26]$ the equilibrium is achieved after 30000 Metropolis steps. (Far from the phase transition the convergence is even faster.) The autocorrelation time for the gauge fields is about 800 Metropolis steps. For the scalar field the autocorrelation time is one order of magnitude smaller. The code was tested in several ways. In particular, some previous results in the $SU(2)$ gauge - Higgs model [1, 9, 10] and the Weinberg - Salam model with frozen radius of the scalar field [8] were reproduced.

4.2 Cold start and Hot start

When the simulation starts from the cold start, the equilibrium is achieved approximately after 30000 Metropolis steps for $\gamma > 0.26$ and $\gamma < 0.257$. (Far from the transition the number of steps needed in order to achieve equilibrium is smaller.) This way we obtain, in particular, the equilibrium at $\gamma = 0.255$. Next, starting from the corresponding configurations the further simulation procedure is applied with the values of γ between 0.256 and 0.264. We call this simulation the hot start simulation¹. The results of these simulations are presented in Fig. 1. In this figure the data of the link part of the action $\frac{1}{4N} \sum_{xy} H_x^+ U_{xy} e^{i\theta_{xy}} H_y$ (that is most sensitive to the transition between the Higgs phase and the symmetric phase) are represented. The squares correspond to the cold start (450000 Metropolis steps)². The crosses correspond to the hot start (350000 Metropolis steps). One can see that the two lines merge together. However, at earlier stages of the simulation the Hysteresis pattern was observed that points to the interval $[0.257, 0.26]$ as to the place of the transition between the two phases. The convergence to the equilibrium in this simulation is rather slow. Our data demonstrate the absence of the two - state signal for $\gamma \in [0.257, 0.26]$.

¹The simulation that begins from the completely disordered configurations converges to the equilibrium much more slow. This is because the Algorithm has to overcome the confinement - deconfinement phase transition due to $U(1)$ and to transfer the configuration with zero β to the configuration with rather high value $\beta = 12$. In practise such a simulation never achieves equilibrium for the number of Metropolis steps up to 200000 on the lattice $8^3 \times 16$. However, we observed the convergence on smaller lattices of sizes up to 6^4 .

²This simulation has required about 480 ours CPU time.

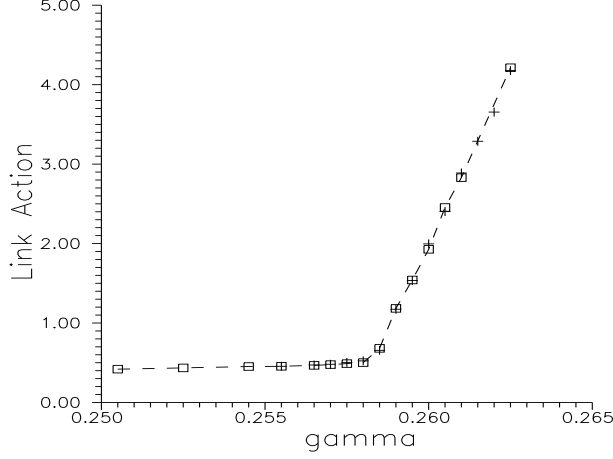


Figure 1: The link part of the action as a function of γ at $\lambda = 0.0025$, $\beta = 12$. Cold start corresponds to squares. Hot start corresponds to crosses. The error bars are about of the same size as the symbols used.

4.3 Ultraviolet Effective potential

In view of (10) we use instead of the ultraviolet potential (1) the expression $U^{u-v}(\phi) = V^{u-v}(\phi) + 3 \log \phi$. (The term $3 \log \phi$ comes from the measure over the scalar doublet that has 4 real components.)

According to the results obtained in [2] at $\gamma \leq \gamma_c$ there is the only minimum of the ultraviolet effective constraint potential U^{u-v} . This is illustrated by Fig. 2. At the same time for $\gamma > \gamma_c$ the ultraviolet potential has the only minimum at nonzero value of ϕ (see also Fig. 2).

Our present numerical results for the ultraviolet potential show that at $\gamma = 0.268$ the best fit to U^{u-v} is

$$U^{u-v}(\phi) = const + 0.83(\phi - 2.75)^2 \quad (12)$$

At the same time the estimate given in Section 3 is (for $m_H = 4\sqrt{\frac{\gamma - \gamma_c^{(0)}}{\gamma}} \sim 1.1$):

$$U^{u-v}(\phi) = const + \frac{\gamma}{2G_{m_H}(0)}(\phi - v)^2 \sim const + 1.06(\phi - 3.8)^2 \quad (13)$$

We observe the 20 percent discrepancy between the measured dispersion and its tree level estimate and even larger discrepancy between the value of

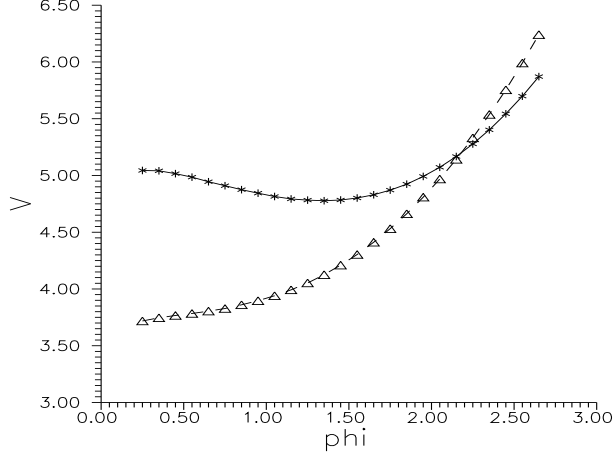


Figure 2: The ultraviolet effective constraint potential at $\gamma = 0.26$ (triangles, dashed line) and $\gamma = 0.262$ (crosses, solid line); $\lambda = 0.0025$, $\beta = 12$. Error bars are about of the same size as the symbols used.

v calculated using the ultraviolet potential and its tree - level estimate. As for the critical value γ_c , its tree level estimate is $\gamma_c^{(0)} = 0.24875$ while the fluctuational region is localized between 0.252 and 0.262

4.4 Infrared Effective Potential UDZ at γ_c

This potential has the only minimum at $\phi = 0$ for $\gamma \leq \gamma_c = 0.26$. At $\gamma > \gamma_c$ the UDZ potential has the nontrivial minima at nonzero values of ϕ (see Fig. 3). Already at $\gamma = 0.262 \sim \gamma_{c2}$ there is the only nontrivial minimum of the potential that is deeper than the other local minima observed.

4.5 Infrared Effective potential UZ and the transition at γ'_c

The infrared potential UZ has the minimum at $\phi = 0$ for $\gamma \leq 0.2575$ (see Fig.4). At $\gamma \geq 0.258$ the given potential has nontrivial minimum at nonzero value of ϕ (Fig.4).

The given results point to $\gamma'_c = 0.25775 \pm 0.00025$ as to the point, where the UZ potential changes its form.

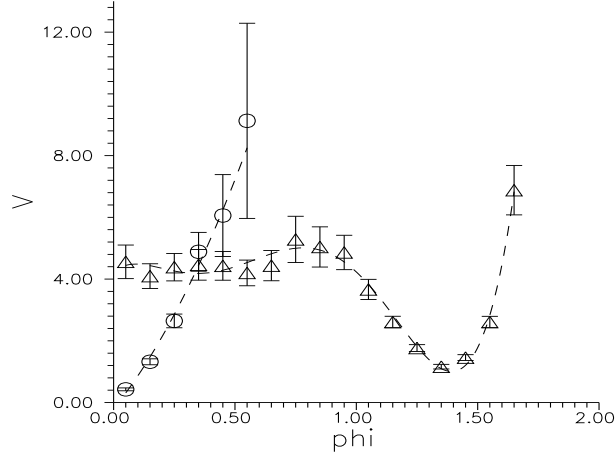


Figure 3: The infrared potential Eq.(2) UDZ at $\gamma = 0.262$ (triangles), and at $\gamma = 0.26$ (circles), $\lambda = 0.0025$, $\beta = 12$.

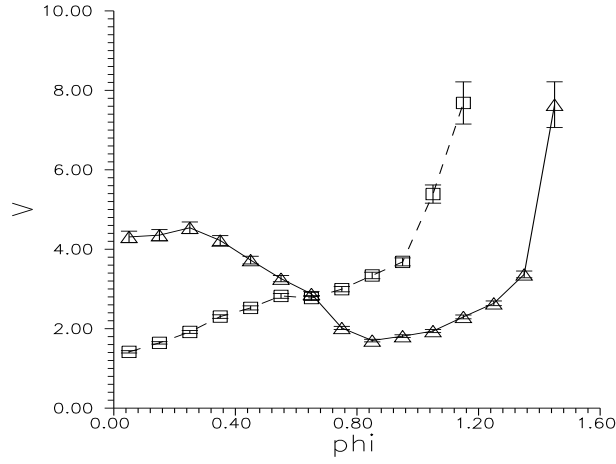


Figure 4: The infrared potential Eq.(2) UZ at $\gamma = 0.2575$ (squares), and $\gamma = 0.258$ (triangles); $\lambda = 0.0025$, $\beta = 12$.

4.6 Z - boson mass from the infrared potential

The present numerical results on the Z - boson mass in the Z - version of Unitary gauge confirm the results of [2]. Nonzero values of Z - boson mass are obtained at $\gamma > \gamma'_c$. At the same time for $\gamma < \gamma'_c$ we observe large statistical errors for the ZZ correlator. Therefore, in this region of the phase diagram the Z - boson mass cannot be calculated and we suppose it vanishes somewhere between $\gamma = 0.25$ and $\gamma = \gamma'_c$.

At $\gamma = 0.268$ the infrared potentials give the value of $v = 2.95 \pm 0.05$ that is to be compared with the mentioned above tree level estimate and the value given by the ultraviolet potential. It is instructive to calculate lattice Z - boson mass using the given value of v and the expression $m_Z = v \sqrt{\frac{\gamma}{\beta \cos^2 \theta_W}}$. Using the value $v = 2.95 \pm 0.05$ we obtain in this way $m'_Z = 0.51 \pm 0.01$. This is to be compared with the value of lattice Z - boson mass reported in [2]: $m_Z = 0.49 \pm 0.01$. That's why at this value of γ the infrared potential gives reasonable estimate for the scalar field condensate. At $\gamma = 0.262$ the infrared potential UDZ gives the value $v_{UDZ} = 1.45 \pm 0.05$ while the UZ potential gives $v_{UZ} = 2.35 \pm 0.05$. These values give $m'_{UDZ} = 0.25 \pm 0.01$ and $m'_{UZ} = 0.40 \pm 0.01$ while from [2] we get $m_Z = 0.29 \pm 0.01$. At the same time expression $m_Z = v \sqrt{\frac{\gamma}{\beta \cos^2 \theta_W}}$ gives zero lattice Z - boson mass at $\gamma = \gamma_c$ (when v is extracted from the UDZ potential) or at $\gamma = \gamma'_c$ (when v is extracted from the UZ potential). The value calculated in [2] differs from zero at γ_c . At γ'_c we cannot calculate m_Z due to large statistical errors. Now we do not insist on the validity of one of the mentioned estimates. Instead we suppose that within the Fluctuational region the definition of the gauge boson masses becomes ambiguous because the usual perturbation theory, most likely, does not work there.

4.7 Renormalized fine structure constant

In the present paper in order to calculate the renormalized fine structure constant $\alpha_R = e^2/4\pi$ (where e is the electric charge) we use the correlator of Polyakov lines for the right-handed external leptons. These lines are placed along the selected direction (called below imaginary "time" direction). The space - like distance between the lines is denoted by R .

$$\mathcal{C}(|\bar{x} - \bar{y}|) = \langle \text{Re} \Pi_t e^{2i\theta_{(\bar{x},t)(\bar{x},t+1)}} \Pi_t e^{-2i\theta_{(\bar{y},t)(\bar{y},t+1)}} \rangle. \quad (14)$$

The potential is extracted from this correlator as follows

$$\mathcal{V}(R) = -\frac{1}{L} \log \mathcal{C}(R) \quad (15)$$

Here L is the size of the lattice in the imaginary "time" direction.

Due to exchange by virtual photons at large enough distances one would expect the appearance of the Coulomb interaction

$$\begin{aligned} \mathcal{V}(r) &= -\alpha_R \mathcal{U}_0(r) + const, \\ \mathcal{U}_0(r) &= -\frac{\pi}{N^3} \sum_{\vec{p} \neq 0} \frac{e^{ip_3 r}}{\sin^2 p_1/2 + \sin^2 p_2/2 + \sin^2 p_3/2} \end{aligned} \quad (16)$$

Here N is the lattice size, $p_i = \frac{2\pi}{L} k_i$, $k_i = 0, \dots, L-1$.

However, at smaller distance the better fit to the potential is given by

$$\begin{aligned} \mathcal{V}(r) &= -\alpha_R [\mathcal{U}_0(r) + \frac{1}{3} \mathcal{U}_m(r)] + const, \\ \mathcal{U}_m(r) &= -\frac{\pi}{N^3} \sum_{\vec{p}} \frac{e^{ip_3 r}}{\sin^2 p_1/2 + \sin^2 p_2/2 + \sin^2 p_3/2 + \text{sh}^2 m/2} \end{aligned} \quad (17)$$

Here exchange by virtual massive Z - bosons is taken into account.

We substitute to (17) the linear fit to the Z - boson mass calculated in [2].

The results are presented in Fig. 5 and are to be compared with the tree level estimate for the fine structure constant $\alpha^{(0)} \sim \frac{1}{151}$ and the 1 - loop approximation (when we assume bare value of α to live at the scale ~ 1 TeV while the renormalized value lives at the Electroweak scale M_Z): $\alpha^{(1)}(M_Z/1 \text{ TeV}) \sim \frac{1}{149.7}$.

Fig. 5 demonstrates that the renormalized fine structure constant calculated in the mentioned above way is close to the one - loop estimate (when the cutoff³ Λ in $\alpha^{(1)}(M_Z/\Lambda)$ is around 1 TeV). This confirms indirectly that the values of the Z - boson mass calculated in [2] are correct.

5 Discussion

In the present paper we have reported the results of our numerical investigation of lattice Weinberg - Salam model at $\beta = 12$, $\lambda = 0.0025$, $\theta_W = 30^\circ$.

³For $\Lambda = 1000$ TeV, for example, we would get the one - loop result $\alpha^{(1)}(M_Z/\Lambda) \sim 1/147$ that seems to deviate already from our numerical results presented in Fig.5.

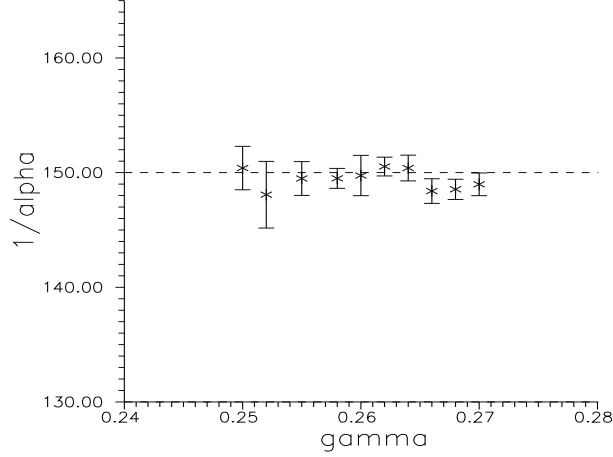


Figure 5: The renormalized fine structure constant as a function of γ at $\lambda = 0.0025$, $\beta = 12$.

For these values of couplings the bare Higgs boson mass is close to 150 GeV near to the transition between the Higgs phase and the symmetric phase. All numerical simulations were performed on rather small lattices (of the size $8^3 \times 16$). However, according to [2] the most important results do not depend on the lattice size for the lattices of the linear size up to 20. Therefore, we feel this appropriate to publish our results considered at the present moment as preliminary.

The effective potential in the model has been calculated in two different ways, after two different gauge fixing procedures are applied. In both cases the scalar doublet has the form $H = \begin{pmatrix} h \\ 0 \end{pmatrix}$, where h is real. However, the signs of h are fixed differently. One of the given effective potentials changes its form at $\gamma_c = 0.26 \pm 0.001$ (at this point the ultraviolet potential calculated in [2] changes its form as well), the other potential changes its form at $\gamma'_c = 0.25775 \pm 0.00025$. Such a pattern is typical for the crossovers: different quantities change their behavior at different points on the phase diagram.

At the present moment we imagine the pattern of the transition as follows. When γ is decreased Z vortices become more and more dense. The first step is the transition to the fluctuational region, where Z - vortices and the Nambu monopoles dominate. This occurs around $\gamma_{c2} \sim 0.262$ (see [2]).

This region may be to some extent similar to the mixed phase of the second order superconductors. This phase of the superconductor appears when the external magnetic field is present and the lattice of Abrikosov vortices appears. These vortices are, in turn, the embryos of the normal phase within the superconducting one. When the magnetic field achieves the second critical value, these vortices overcome the repulsive forces and are transformed into the homogenous symmetric phase. In our case the analogue of this phenomenon may occur at the final step of the transition, where the lattice Z - boson mass vanishes.

According to our results no signs of the two - state signal are found. It is worth mentioning that in classical second order superconductors the transition to the normal phase is usually thought of as the second order phase transition [11]. Lattice simulations [12], in turn, indicate that the transition is a crossover in lattice Ginzburg - Landau Model (when it describes the second order superconductors)⁴. Our data, certainly, show that on the lattice $8^3 \times 16$ the transition in the lattice Weinberg - Salam model is a crossover. This follows from the fact that different observables change their behavior at different points on the phase diagram. In particular, the infrared effective potentials UDZ and DZ change their form at different points. At the same time far from the transition both potentials give the same value of the scalar field condensate. In [2] lattice masses were calculated for $\gamma > \gamma'_c$. At $\gamma < \gamma'_c$ statistical errors do not allow to estimate Z - boson mass. Therefore, we do not exclude at the present moment, that the second order phase transition may appear on the lattices of larger sizes somewhere close to γ'_c . At the same time, it is very unlikely, that the first order phase transition may appear on the larger lattices. To our opinion, the step - like change of the entropy would manifest itself already on the lattice $8^3 \times 16$ if it occurs on an infinite lattice. In addition, we know that there is no first order phase transition in Abelian Higgs Model at $M_H > M_Z$. In lattice Weinberg- Salam model the crossover seems to us the preferred possibility.

The detailed analysis of the considered phase transition is, therefore, still to be performed in order to understand its physics. In particular, it would be very important to repeat our calculations on the larger lattices. Our present results on the infrared effective potentials obtained on the lattice $8^3 \times 16$ can be considered as a starting point of such an investigation.

It is also important to investigate more carefully the possible relation-

⁴In the 4D AHM, however, the situation is not so clear (see, for example, [13]).

ship between the fluctuational region in the Electroweak theory and the mixed phase of the superconductors. The possibility to approach continuum physics within this region of the phase diagram is an important question as well. At a first look in the fluctuational region the Z - vortices and the Nambu monopoles must contribute to the physical observables. This makes it impossible to use only the conventional perturbation expansion around the trivial vacuum $h = \text{const}$. The reason is that this expansion ignores the given topological objects (at least, on the level of the the first terms of the loop expansion). Therefore we suppose that this region cannot serve as a source of the conventional continuum Electroweak physics due to the Z - vortices and Nambu monopoles that dominate there. It is worth mentioning, however, that our calculation of the renormalized fine structure constant within this region shows that the resulting values of α_R are surprisingly close to the one - loop perturbative result $\alpha^{(1)}(M_Z/\Lambda)$ when the cutoff Λ is of the order of 1 TeV.

Finally, we would like to notice that the methods used (including the calculation of the infrared effective potential) can be applied to the investigation of the finite temperature Electroweak phase transition.

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